Online Supplemental Appendix

SA1 Extensions in Section 4.2 of the paper

SA1.1 Non-uniform distribution of $F(\gamma)$

In the basic model analyzed in Section 3 of the paper, we have assumed that $F(\gamma)$ follows a uniform distribution for tractability. In this section, we extend the model to non-uniform distribution. Our focus is on whether the equilibrium can jump discretely from $s^* = 0$ to $s^* > 0$ as c decreases so that the results in Proposition 3 and Corollary 3 are robust.

When d is exogenous, as evident from the proof of Proposition 1 in the Appendix of the paper, the qualitative results of Proposition 1 does not rely on uniform distribution. When d is endogenous, following the similar analysis that leads to Proposition 2, we can show that, under general distribution of $F(\gamma)$, $s^* = 0$ when c is sufficiently large and $s^* > 0$ when $c = 0$. In addition, a change from $s^* = 0$ to $s^* > 0$ must accompany with the equilibrium condition switching from IC_H binding to IC_L binding so that the change in s^* is finite. Hence, a discrete jump from $s^* = 0$ to $s^* > 0$ must occur when c decreases under a general distribution of $F(\gamma)$. As shown in the proof of Proposition 3 and Corollary 3, the results hold regardless of the functional form of $F(\gamma)$ as long as a jump from $s^* = 0$ to $s^* > 0$ occurs when c decreases and $s^* = 0$ is associated with the binding of IC_H and $s^* > 0$ is associated with the binding of IC_L. Therefore, the results in Proposition 3 and Corollary 3 still hold qualitatively under a general distribution of $F(\gamma)$.

With a general distribution of $F(\gamma)$, however, we are not able to rule out the possibility of multiple switches between case 1 and case 2 in Proposition 2 when c decreases without imposing further restrictions on $F(\gamma)$. In a more concrete example, we consider normal distribution $N\left(\frac{1}{2}, \frac{1}{36}\right)$, truncated to $[0, 1]$, and $\alpha = \frac{1}{3}$, $\beta = \frac{1}{2}$, $v = 1$, $k = 1$. The qualitative results in Proposition 3 and Corollary 3 are confirmed.

SA1.2 Imperfect knowledge about ad costs

In the paper, we have assumed that consumers have perfect knowledge of the ad-production cost s, after seeing the ad. In practice, however, consumers' knowledge may be imperfect. To take this into account, we assume that consumers perceive the ad-production cost as δs, where $\delta > 0$ can be above or below 1. Advertisers are aware of consumers' mistakes. Next, we show that our main results remain robust.

Let s^e denote the expectation consumers form about s. $s^e = \delta s^*$ must hold in the equilibrium due to 'rational' expectation. Firm i, $i = L, H$, faces IC_i constraint. Further let IC_i^e denote the (incorrectly) expected constraint by consumers.

The relevant IC constraints are given by,

$$F(\tilde{\gamma})[(1-\beta)v-d]-\delta\cdot s\leq 0, \quad (IC_L^e)$$

$$F(\tilde{\gamma})[(1-\beta)v-d]-s\leq 0, \quad (IC_L)$$

$$F(\tilde{\gamma})[(1-\beta)(2v)-d]\geq 0. \quad (IC_H)$$

Marginal consumer, who is indifferent between blocking ad or not, is given by

$$\alpha\cdot[\gamma-h(\delta s)]=c\Rightarrow\tilde{\gamma}=\frac{c}{\alpha}+h(\delta s).$$

Relative to IC_L, IC_L^e is easier to satisfy if and only if $\delta>1$. We look for a separating equilibrium that satisfies the Intuitive Criterion with $\mu(\delta s,p_1)=1$ if $\delta s\geq\delta s^*$ and $p_1=v$, and $\mu(\delta s,p_1)=0$ if $\delta s<\delta s^*$ and/or $p_1\neq v$. Intuitively, there are two types of equilibrium. In type 1 equilibrium $s^*>0$. Then IC_L^e must be binding. In type 2 equilibrium, $s^*=0$. Then IC_H must be binding.

First, suppose that $\delta>1$

Case 1: IC_L^e binding with $s^*>0$

Binding IC_L^e implies that

$$d\cdot F(\tilde{\gamma})=F(\tilde{\gamma})(1-\beta)v-\delta\cdot s.$$

With IC_L^e binding and $\delta>1$, IC_L will be violated. So L-type firm actually will advertise, together with H-type firm. However, uninformed consumers incorrectly believe that only the H-type firm will advertise.

Platform's profit is given by

$$D_1=dF(\tilde{\gamma})=F(\tilde{\gamma})(1-\beta)v-\delta\cdot s.$$

Solving the FOC, we can obtain

$$\delta s^*=\left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2, \quad h(\delta s^*)=k\sqrt{\delta s^*}=\frac{(1-\beta)k^2v}{2\bar{\gamma}}.$$

Then

$$D_1=(1-\beta)\frac{c}{\alpha\bar{\gamma}}\cdot v+\left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2.$$

Case 2: IC_H binding with $s^*=0$

Binding IC_H implies that

$$d\cdot F(\tilde{\gamma})=(1-\beta)F(\tilde{\gamma})(2v).$$

With $s^* = 0$, IC_L^e and IC_L coincide. When IC_H is binding, both IC_L and IC_L^e are slack. Only H-type firm will advertise, and platform's profit is

$$D_2 = \alpha \cdot d \cdot F(\tilde{\gamma}) = (1-\beta)\frac{c}{\bar{\gamma}} \cdot (2v).$$

Note that $\frac{dD_2}{dc} / \frac{dD_1}{dc} = 2\alpha$. If $\alpha \le \frac{1}{2}$, then $D_2 < D_1$ always holds. If $\alpha > \frac{1}{2}$, then single-crossing condition holds. Let $\bar{c}$ be defined by $D_1(\bar{c}) = D_2(\bar{c})$. Then $D_1 > D_2$ if and only if $c < \bar{c}$.

$$D_1 = D_2 \Rightarrow (1-\beta)\frac{c}{\bar{\gamma}} \cdot \left(2 - \frac{1}{\alpha}\right) v = \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2$$

$$\Rightarrow \bar{c} = \frac{(1-\beta)k^2 v}{4\left(2 - \frac{1}{\alpha}\right)\bar{\gamma}}.$$

Therefore, when c decreases, the equilibrium jumps from case 2 ($s^* = 0$) to case 1 ($s^* > 0$) exactly once, at $c = \bar{c}$.

Next, suppose $\delta < 1$.

Case 1: IC_L^e binding with $s^* > 0$

In this case, platform chooses d^* (and in turn s^*) to maximize its profit D_1. Solution to the FOC leads to the same expressions of d^* and s^* as when $\delta > 1$.

With IC_L^e binding, IC_L is slack. IC_H is satisfied if and only if

$$\delta \ge \frac{(1-\beta)v - d^*}{(1-\beta)(2v) - d^*} \equiv \delta_1 \in (0,1).$$

When $\delta < \delta_1$, H-type firm will not advertise. With no firm advertising, this cannot be optimal so there is no type 1 equilibrium (with $s^* > 0$). Next, suppose that $\delta \ge \delta_1$.

Binding IC_L^e implies that

$$d \cdot F(\tilde{\gamma}) = F(\tilde{\gamma})(1-\beta)v - \delta \cdot s.$$

Only H-type firm will advertise so platform's profit is

$$D_1 = \alpha \cdot dF(\tilde{\gamma}) = \alpha \cdot [F(\tilde{\gamma})(1-\beta)v - \delta \cdot s].$$

Solving the FOC, we can obtain

$$\delta s^* = \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2, \quad h(\delta s^*) = k\sqrt{\delta s^*} = \frac{(1-\beta)k^2 v}{2\bar{\gamma}}.$$

Then

$$D_1 = (1-\beta)\frac{c}{\bar{\gamma}} \cdot v + \alpha \cdot \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2.$$

Case 2: IC_H binding with $s^* = 0$

Since $s^* = 0$, IC_L^e and IC_L coincide. With IC_H binding, both IC_L^e and IC_L are slack. Binding IC_H implies that

$$d \cdot F(\tilde{\gamma}) = (1-\beta)F(\tilde{\gamma})(2v).$$

Platform's profit is

$$D_2 = \alpha \cdot d \cdot F(\tilde{\gamma}) = (1-\beta)\frac{c}{\bar{\gamma}} \cdot (2v).$$

Note that $\frac{dD_2}{dc} = 2\frac{dD_1}{dc}$ so single-crossing condition holds. Let $\bar{c}$ be defined by $D_1(\bar{c}) = D_2(\bar{c})$. Then $D_1 > D_2$ if and only if $c < \bar{c}$.

$$D_1 = D_2 \Rightarrow (1-\beta)\frac{c}{\bar{\gamma}} \cdot v = \alpha \cdot \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2$$

$$\Rightarrow \bar{c} = \frac{\alpha(1-\beta)k^2 v}{4\bar{\gamma}}.$$

Therefore, when c decreases, the equilibrium jumps from case 2 ($s^* = 0$) to case 1 ($s^* > 0$) exactly once, at $c = \bar{c}$.■

SA1.3 Multiple ad quantity

In our basic model, we assume that when a firm advertises, the ad would reach all consumers who do not block ad. In this case, the firm only needs to advertise once. In reality, a single ad placement may not be able to reach all the consumers. This creates an interesting tension. To reach more consumers, the ad needs to be placed in multiple slots, but then some consumers may be reached multiple times with repeated payment of the unit ad-distribution cost d.

To allow multiple ad quantities, let n be the number of placements of an ad and assume that the ad blocking decision cannot be changed after a consumer sees some placements of the ad but not the other placements of it. If a single ad placement can reach all consumers who do not block ads, it is easy to see that it is optimal for the advertiser (H-type firm) to set $n = 1$ in separating equilibrium. This is because any spending on ad distribution beyond $n = 1$ is strictly dominated by spending the same amount in ad-production cost, s, which increases the total demand through $h(s)$. If a single ad placement does not reach all consumers, without loss of generality, we consider the case when there are two ad slots and $n \in \{0, 1, 2\}$. If $n = 0$, the firm does not advertise. If $n = 1$, the firm randomly chooses one of the two slots to advertise. We further assume that a $\frac{\theta}{2}$ proportion of consumers who do not block ads visit each of the two ad slots exclusively, with $0 < \theta < 1$. The remaining $1 - \theta$ proportion of consumers visit both slots. For simplicity, we focus on two limiting cases: (i) when θ is sufficiently small (i.e., $\theta \to 0^+$) and (ii) when θ is sufficiently large (i.e., $\theta \to 1^-$). In both cases, we can show that the results corresponding to Proposition 3 and Corollary 3 still hold (for inelastic demand), and similarly for elastic demand. In particular, when θ is sufficiently small, we have $n^H = 1$ and $n^L = 0$ for the H-type and L-type firms respectively, in the separating equilibrium.

When θ is sufficiently large, however, we have $n^H = 2$ and $n^L = 0$ for the H-type and L-type firms respectively. In the equilibrium with $n^H = 2$, each ad placement has less than full coverage and some consumers ($1 - \theta$ proportion) are exposed to both ad placements so that the H-type firm pays $2d$ to the ad platform for each of those consumers.

Let (n, s^*, p_1) denote the advertising firm's choice. How uninformed consumers assign beliefs of firm type when seeing s^* and p_1 is the same as in the basic model. But beliefs for n worth some discussion. In particular, consumers may expect $n = 1$ in the equilibrium but actually observe $n = 2$, or the other way around. This "inconsistency" can only happen to the fraction $1 - \theta$ of consumers who visit both ad slots. Intuitively, H-type firm has more to gain relative to L-type firm when rasing $n = 1$ to $n = 2$. Therefore, we assign the following off-equilibrium belief. If consumers expect $n = 1$ in equilibrium but observe $n = 2$, they will believe the firm is H-type. However, if consumers expect $n = 2$ but observe $n = 1$, they will believe the firm is L-type.

SA1.3.1 Inelastic demand

We start with the case of inelastic demand.

Suppose that θ is sufficiently small ($\theta \to 0^+$).

Most consumers visit both ad slots so advertising at either slot will allow the advertiser to reach them already. Thus in the equilibrium advertiser should display ad at one ad slot only ($n^H = 1$ and $n^L = 0$).

The marginal uninformed consumers can be derived as follows.

$$\text{Those who visit one ad slot:} \quad \alpha \cdot \frac{1}{2} \cdot (\gamma - h(s)) = c \Rightarrow \tilde{\gamma}_1 = \frac{2c}{\alpha} + h(s).$$

$$\text{Those who visit both ad slots:} \quad \alpha \cdot (\gamma - h(s)) = c \Rightarrow \tilde{\gamma}_2 = \frac{c}{\alpha} + h(s).$$

Next, we analyze advertiser and platform behavior. There are two cases, depending on whether $s^* > 0$ in the equilibrium.

<u>Case 1: IC_L binding with $s^* > 0$</u>

In this case, IC_L is binding and IC_H is slack.

A binding IC_L suggests that the L-type firm has no incentive to deviate from $n^L = 0$ to $n^L = 1$ or $n^L = 2$.

On the other hand, since IC_H slack, the H-type firm has no incentive to deviate from $n^H = 1$ to $n^H = 0$. It has no incentive to deviate to $n^H = 2$ either because with $n^H = 1$ it already reaches almost all of the consumers.

Next, we consider the platform. With IC_L binding, platform profit is,

$$
\begin{aligned}
D_1(n=1) &= \alpha \cdot \left[(1-\beta)\cdot\left(\frac{\theta}{2}\cdot F(\tilde{\gamma}_1)+(1-\theta)\cdot F(\tilde{\gamma}_2)\right)\cdot(v)-s\right] \\
&= \alpha\cdot(1-\beta)\cdot\left[\frac{\theta}{2}\cdot\left(\frac{2c}{\alpha\bar{\gamma}}+h(s)\right)+(1-\theta)\cdot\left(\frac{c}{\alpha\bar{\gamma}}+h(s)\right)\right]\cdot(v)-\alpha\cdot s \\
&= (1-\beta)\cdot\frac{c}{\bar{\gamma}}\cdot v+\frac{\alpha(1-\beta)(2-\theta)v}{2\bar{\gamma}}\cdot h(s)-\alpha\cdot s.
\end{aligned}
$$

Solving the FOC, we can obtain

$$
s^* = \left(\frac{(1-\beta)(2-\theta)kv}{4\bar{\gamma}}\right)^2, \quad h(s^*) = \frac{(1-\beta)(2-\theta)k^2 v}{4\bar{\gamma}}.
$$

Then

$$
D_1(n=1) = (1-\beta)\cdot\frac{c}{\bar{\gamma}}\cdot v+\alpha\cdot\left(\frac{(1-\beta)(2-\theta)kv}{4\bar{\gamma}}\right)^2.
$$

Case 2: IC_H binding with $s^*=0$

With IC_H binding, $\pi_H(n^H=1)=\pi_H^{dev}(n^H=0)$ so H-type firm has no incentive to deviate to $n^H=0$. It has no incentive to deviate to $n^H=2$ either because it already reaches most of the consumers with $n^H=1$. Similarly, the L-type firm has no incentive to deviate from $n^L=0$ (not advertising) to $n^L=1$ or $n^L=2$.

Next, we derive platform profit. Note that with $n^H=1$, the ad will reach all consumers who visit both ad slots, but will reach only half of the consumers who visit one ad slot.

$$
\begin{aligned}
D_2(n=1) &= \alpha\cdot(1-\beta)\left[\frac{\theta}{2}\cdot F(\tilde{\gamma}_1)+(1-\theta)\cdot F(\tilde{\gamma}_2)\right]\cdot(2v) \\
&= \alpha\cdot(1-\beta)\left[\frac{\theta}{2}\cdot\frac{2c}{\alpha\bar{\gamma}}+(1-\theta)\cdot\frac{c}{\alpha\bar{\gamma}}\right]\cdot(2v) \\
&= (1-\beta)\cdot\frac{c}{\bar{\gamma}}\cdot(2v).
\end{aligned}
$$

It can be easily verified that $\frac{dD_2(n=1)}{dc}=2\frac{dD_1(n=1)}{dc}$ so single-crossing condition is satisfied. There exists a unique $\bar{c}$ such that $D_2(n=1)>D_1(n=1)$ if and only if $c>\bar{c}$.

Suppose that θ is sufficiently large ($\theta\to 1^-$).

Most consumers visit only one ad slot. In the equilibrium the advertiser must advertise at both ad slots ($n^H=2$, $n^L=0$).

We first derive the marginal uninformed consumers:

$$
\text{Those who visit one slot:}\quad \alpha\cdot(\gamma-h(s))=c\Rightarrow\tilde{\gamma}_1=\frac{c}{\alpha}+h(s).
$$

$$
\text{Those who visit both slots:}\quad \alpha\cdot 2\cdot(\gamma-h(s))=c\Rightarrow\tilde{\gamma}_2=\frac{c}{2\alpha}+h(s).
$$

Next, we analyze advertiser and platform behavior. There are two cases, depending on whether $s^* > 0$ in the equilibrium.

Case 1: IC_L binding with $s^* > 0$

With IC_L binding, $\pi_L(n^L = 0) = \pi_L^{dev}(n^L = 2)$ so L-type firm has no incentive to deviate from $n^L = 0$ to $n^L = 2$. It does not want to deviate to $n^L = 1$ either since $\pi_L^{dev}(n^L = 2) > \pi_L^{dev}(n^L = 1)$. This is because, going from $n^L = 1$ to $n^L = 2$, sales to uninformed consumers and ad distribution cost both double, but the ad production cost stays the same.

Since IC_L is binding, IC_H must be slack. So H-type firm has no incentive to deviate from $n^H = 2$ to $n^H = 0$. Deviating to $n^H = 1$ will also reduce its profit.

With IC_L binding, platform profit is,

$$\begin{aligned} D_1(n=2) &= \alpha \cdot [(1-\beta) \cdot (\theta \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)) \cdot (v) - s] \\ &= \alpha \cdot (1-\beta) \cdot \left[\theta \cdot \left(\frac{c}{\alpha\bar{\gamma}} + h(s)\right) + (1-\theta) \cdot \left(\frac{c}{2\alpha\bar{\gamma}} + h(s)\right)\right] \cdot (v) - \alpha \cdot s \\ &= (1-\beta)\left(\theta + \frac{1-\theta}{2}\right) \cdot \frac{c}{\bar{\gamma}} \cdot (v) + \frac{\alpha(1-\beta)v}{\bar{\gamma}} \cdot h(s) - \alpha \cdot s. \end{aligned}$$

Solving the FOC, we can obtain

$$s^* = \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2, \quad h(s^*) = \frac{(1-\beta)k^2 v}{2\bar{\gamma}}.$$

Then

$$D_1(n=2) = (1-\beta)\left(\theta + \frac{1-\theta}{2}\right) \cdot \frac{c}{\bar{\gamma}} \cdot (v) + \alpha \cdot \left(\frac{(1-\beta)kv}{2\bar{\gamma}}\right)^2.$$

Case 2: IC_H binding with $s^* = 0$

With IC_H binding, we have $\pi_H(n = 2) = \pi_H^{dev}(n = 0)$. It can also be shown that $\pi_H^{dev}(n = 1) > \pi_H(n = 2)$. This is because, going from $n = 2$ to $n = 1$, ad-distribution cost is reduced by exactly half while sales from uninformed consumers is reduced by slightly less than half (due to the consumers who visit both ad slots). Then binding IC_H, in the sense when there is only one ad slot available, does not hold in the equilibrium anymore. Intuitively, the unit ad-distribution cost d must go down slightly so $\pi_H(n = 2) = \pi_H^{dev}(n = 1)$ (and both are above $\pi_H^{dev}(n = 0)$). Since $\theta \to 1^-$, this reduction in d is infinitesimal. Then H-type firm has no incentive to deviate to $n = 1$ or $n = 0$.

It can also be shown that L-type firm has no incentive to deviate from $n = 0$ to $n = 1$ or $n = 2$.

Since $\pi_H(n = 2) = \pi_H^{dev}(n = 1)$, we have

$$\begin{aligned} &(1-\beta)\left[\theta \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right] \cdot (2v) - d \cdot \left[\theta \cdot F(\tilde{\gamma}_1) + 2(1-\theta) \cdot F(\tilde{\gamma}_2)\right] \\ = &(1-\beta)\left[\frac{\theta}{2} \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right] \cdot (2v) - d \cdot \left[\frac{\theta}{2} \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right] \end{aligned}$$

$$\Rightarrow d \cdot \left[\frac{\theta}{2} \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right] = (1-\beta) \cdot \frac{\theta}{2} \cdot F(\tilde{\gamma}_1) \cdot (2v).$$

Then platform profit is

$$\begin{aligned} D_2(n=2) &= \alpha \cdot d \cdot [\theta \cdot F(\tilde{\gamma}_1) + 2(1-\theta) \cdot F(\tilde{\gamma}_2)] \\ &= \alpha \cdot (1-\beta)\theta \cdot F(\tilde{\gamma}_1) \cdot (2v) \\ &= (1-\beta)\theta \frac{c}{\bar{\gamma}} \cdot (2v). \end{aligned}$$

It can be easily verified that when $\theta \to 1^-$, $\frac{dD_2(n=2)}{dc} \approx 2 \cdot \frac{dD_1(n=2)}{dc}$ so single-crossing condition holds. There exists a unique $\bar{c}$ such that $D_2(n=2) > D_1(n=2)$ if and only if $c > \bar{c}$.

SA1.3.2 Elastic demand

Next, we consider the case of elastic demand. The analysis is similar to that in the case of inelastic demand, except that the marginal uninformed consumer will look different.

Suppose that θ is sufficiently small ($\theta \to 0^+$)

Then most consumers visit both ad slots. Intuitively $n^H = 1$ must hold in the equilibrium and no firm (high or L-type) will ever deviate to $n = 2$.

We first derive the marginal uninformed consumers.

$$\text{Those who visit one slot:} \quad \alpha \cdot \frac{1}{2} \cdot \left[(\gamma - h(s)) - v^2\right] = c \Rightarrow \tilde{\gamma}_1 = v^2 + \frac{2c}{\alpha} + h(s).$$

$$\text{Those who visit both slots:} \quad \alpha \cdot \left[(\gamma - h(s)) - v^2\right] = c \Rightarrow \tilde{\gamma}_2 = v^2 + \frac{c}{\alpha} + h(s).$$

Next, we analyze advertiser and platform behavior. There are two cases, depending on whether $s^* > 0$.

Case 1: IC_L binding with $s^* > 0$

With IC_L binding and IC_H slack, L-type firm is indifferent between $n = 0$ and $n = 1$, while the H-type firm prefers $n = 1$ to $n = 0$.

The platform profit is,

$$\begin{aligned} D_1(n=1) &= \alpha \cdot \left[(1-\beta) \cdot \left(\frac{\theta}{2} \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right) \cdot (v^2) - s\right] \\ &= \alpha \cdot \frac{1-\beta}{\bar{\gamma}} \cdot \left[\frac{\theta}{2} \cdot \left(v^2 + \frac{2c}{\alpha} + h(s)\right) + (1-\theta) \cdot \left(v^2 + \frac{c}{\alpha} + h(s)\right)\right] \cdot (v^2) - \alpha \cdot s \\ &= \alpha \cdot \frac{(1-\beta)(2-\theta)v^2}{2\bar{\gamma}} \cdot (v^2) + (1-\beta) \cdot \frac{c}{\bar{\gamma}} \cdot v^2 + \alpha \cdot \frac{(1-\beta)(2-\theta)v^2}{2\bar{\gamma}} \cdot h(s) - \alpha \cdot s. \end{aligned}$$

Solving the FOC, we can obtain

$$s^* = \left(\frac{(1-\beta)(2-\theta)kv^2}{4\bar{\gamma}}\right)^2, \quad h(s^*) = \frac{(1-\beta)(2-\theta)k^2v^2}{4\bar{\gamma}}.$$

Then

$$D_1(n=1) = \alpha \cdot \frac{(1-\beta)(2-\theta)v^2}{2\bar{\gamma}} \cdot (v^2) + (1-\beta) \cdot \frac{c}{\bar{\gamma}} \cdot v^2 + \alpha \cdot \left(\frac{(1-\beta)(2-\theta)kv^2}{4\bar{\gamma}}\right)^2.$$

Case 2: IC_H binding with $s^* = 0$

With IC_H binding, the H-type firm is indifferent between $n = 0$ and $n = 1$, while the L-type firm prefers $n = 0$ to $n = 1$.

Platform profit is

$$\begin{aligned} D_2(n=1) &= \alpha \cdot (1-\beta)\left[\frac{\theta}{2} \cdot F(\tilde{\gamma}_1) + (1-\theta) \cdot F(\tilde{\gamma}_2)\right] \cdot (2v^2) \\ &= \alpha \cdot \frac{1-\beta}{\bar{\gamma}}\left[\frac{\theta}{2} \cdot \left(v^2 + \frac{2c}{\alpha}\right) + (1-\theta) \cdot \left(v^2 + \frac{c}{\alpha}\right)\right] \cdot (2v^2) \\ &= \alpha \cdot \frac{(1-\beta)(2-\theta)}{2\bar{\gamma}} \cdot v^2 \cdot (2v^2) + (1-\beta) \cdot \frac{c}{\bar{\gamma}} \cdot (2v^2). \end{aligned}$$

It can be easily verified that $\frac{dD_2(n=1)}{dc} = 2 \cdot \frac{dD_1(n=1)}{dc}$ so single-crossing condition is satisfied. There exists a unique $\bar{c}$ such that

$$D_2(n=1) > D_1(n=1) \Rightarrow$$
$$\alpha \cdot \frac{(1-\beta)(2-\theta)v^2}{2\bar{\gamma}} \cdot (v^2) + (1-\beta) \cdot \frac{\bar{c}}{\bar{\gamma}} \cdot v^2 = \alpha \cdot \left(\frac{(1-\beta)(2-\theta)kv^2}{4\bar{\gamma}}\right)^2.$$

The resulting $\bar{c} > 0$ if k is above certain threshold which also ensures that

$$D_1(c=0) > D_2(c=0) \Leftrightarrow \alpha \cdot \frac{(1-\beta)(2-\theta)v^2}{2\bar{\gamma}} \cdot (v^2) < \alpha \cdot \left(\frac{(1-\beta)(2-\theta)kv^2}{4\bar{\gamma}}\right)^2.$$

Suppose that θ is sufficiently large ($\theta \to 1^-$)

Then most consumers visit only one ad slot. The equilibrium must have $n = 2$.

We first derive the marginal uninformed consumers.

$$\text{Those who visit one slot:} \quad \alpha \cdot (\gamma - h(s)) - \alpha \cdot v^2 = c \Rightarrow \tilde{\gamma}_1 = v^2 + \frac{c}{\alpha} + h(s).$$

$$\text{Those who visit both slots:} \quad \alpha \cdot 2 \cdot (\gamma - h(s)) - \alpha v^2 = c \Rightarrow \tilde{\gamma}_2 = \frac{v^2}{2} + \frac{c}{2\alpha} + h(s).$$

Next, we analyze advertiser and platform behavior. There are two cases, depending on whether $s^* > 0$.

Case 1: IC_L binding with $s^* > 0$

Similar to the case of inelastic demand, binding IC_L means that the L-type firm is indifferent between $n = 0$ and $n = 2$. It has no incentive to deviate from $n = 0$ to $n = 2$ either because

$\pi_L^{dev}(n=2) > \pi_L^{dev}(n=1)$. H-type firm must strictly prefer $n=2$ to $n=0$. It has no incentive to deviate to $n=1$ either, because $\pi_H(n=2) > \pi_H^{dev}(n=1)$.

Platform profit is,

$$\begin{aligned} D_1(n=2) &= \alpha \cdot \left[(1-\beta)\cdot(\theta\cdot F(\tilde{\gamma}_1) + (1-\theta)\cdot F(\tilde{\gamma}_2))\cdot(v^2) - s\right] \\ &= \alpha \cdot \frac{1-\beta}{\bar{\gamma}} \cdot \left[\theta\cdot\left(v^2 + \frac{c}{\alpha} + h(s)\right) + (1-\theta)\cdot\left(\frac{v^2}{2} + \frac{c}{2\alpha} + h(s)\right)\right]\cdot(v^2) - \alpha\cdot s \\ &= \frac{\alpha(1-\beta)}{\bar{\gamma}} \cdot \left(\theta + \frac{1-\theta}{2}\right)\cdot\left(v^2 + \frac{c}{\alpha}\right)\cdot(v^2) + \frac{\alpha(1-\beta)v^2}{\bar{\gamma}}\cdot h(s) - \alpha\cdot s. \end{aligned}$$

Solving the FOC, we can obtain

$$s^* = \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2, \quad h(s^*) = \frac{(1-\beta)k^2v^2}{2\bar{\gamma}}.$$

Then

$$D_1(n=2) = \frac{\alpha(1-\beta)}{\bar{\gamma}}\cdot\left(\theta + \frac{1-\theta}{2}\right)\cdot\left(v^2 + \frac{c}{\alpha}\right)\cdot(v^2) + \alpha\cdot\left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$

<u>Case 2: IC_H binding with $s^*=0$</u>

Similar to the case of inelastic demand, d cannot be so high that $\pi_H(n=2) = \pi_H^{dev}(n=0)$. Instead, it has to go down slightly so $\pi_H(n=2) = \pi_H^{dev}(n=1)$.

Then

$$\begin{aligned} &(1-\beta)\left[\theta\cdot F(\tilde{\gamma}_1) + (1-\theta)\cdot F(\tilde{\gamma}_2)\right]\cdot(2v^2) - d\cdot\left[\theta\cdot F(\tilde{\gamma}_1) + 2(1-\theta)\cdot F(\tilde{\gamma}_2)\right] \\ =\ &(1-\beta)\left[\frac{\theta}{2}\cdot F(\tilde{\gamma}_1) + (1-\theta)\cdot F(\tilde{\gamma}_2)\right]\cdot(2v^2) - d\cdot\left[\frac{\theta}{2}\cdot F(\tilde{\gamma}_1) + (1-\theta)\cdot F(\tilde{\gamma}_2)\right] \\ &\Rightarrow (1-\beta)\cdot\frac{\theta}{2}\cdot F(\tilde{\gamma}_1)\cdot(2v^2) = d\cdot\left[\frac{\theta}{2}\cdot F(\tilde{\gamma}_1) + (1-\theta)\cdot F(\tilde{\gamma}_2)\right]. \end{aligned}$$

L-type firm has no incentive to deviate from $n=0$ to $n=1$ or $n=2$.

Platform profit is

$$\begin{aligned} D_2(n=2) &= \alpha\cdot d\cdot\left[\theta\cdot F(\tilde{\gamma}_1) + 2(1-\theta)\cdot F(\tilde{\gamma}_2)\right] \\ &= \alpha\cdot(1-\beta)\theta\cdot F(\tilde{\gamma}_1)\cdot(2v^2) \\ &= \frac{\alpha(1-\beta)}{\bar{\gamma}}\cdot\theta\cdot\left(v^2 + \frac{c}{\alpha}\right)\cdot(2v^2). \end{aligned}$$

It can be easily verified that when $\theta \to 1^-$,

$$\frac{dD_2(n=2)}{dc} \approx 2\cdot\frac{dD_1(n=2)}{dc},$$

so single-crossing condition is satisfied. There exists a unique $\bar{c}$ such that

$$D_2(n=1) > D_1(n=1) \Rightarrow$$

$$\frac{\alpha(1-\beta)}{\bar{\gamma}} \cdot \theta \cdot \left(v^2 + \frac{c}{\alpha}\right) \cdot (2v^2) = \frac{\alpha(1-\beta)}{\bar{\gamma}} \cdot \left(\theta + \frac{1-\theta}{2}\right) \cdot \left(v^2 + \frac{c}{\alpha}\right) \cdot (v^2) + \alpha \cdot \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$

Evaluated at $\theta = 1$, we have

$$\frac{\alpha(1-\beta)}{\bar{\gamma}} \cdot \left(v^2 + \frac{\bar{c}}{\alpha}\right) \cdot (v^2) = \alpha \cdot \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$

The resulting $\bar{c} > 0$ if k is above certain threshold which also ensures that

$$D_1(c=0) > D_2(c=0) \Leftrightarrow \frac{\alpha(1-\beta)}{\bar{\gamma}} \cdot (v^2) \cdot (v^2) < \alpha \cdot \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$

SA1.4 Ad skipping vs. ad blocking

In this section, we further consider and compare two ways for consumers to avoid ads: ad blocking and ad skipping. With ad blocking, which we have explored in the paper, consumers do not encounter any ad by the firm. With ad skipping, however, consumers may decide whether to skip an ad at any time after seeing a part of it. We denote the cost of ad block as c^b and the cost of ad skipping as c^s, and consider the following three ad skipping options that a consumer may adopt. We focus on the case of endogenous d. As we will show later, when demand is inelastic, platform profit is always higher with ad blocking than with ad skipping. Thus we consider elastic demand.

- Option 1: The consumer skips an ad right away. In this case, she remains unaware of the product and incurs no nuisance cost, similar to ad blocking.
- Option 2: The consumer may spend a few seconds watching the ad, which allows her to learn s, with a fraction $t \in (0,1)$ of the nuisance cost. She then skips the rest of the ad at cost c^s. Note that this option is unavailable under ad blocking.
- Option 3: The consumer watches the entire ad and incurs the full nuisance cost.

We are mainly interested in analyzing whether the platform will prefer ad blocking to ad skipping and whether our main results in Section 3 are robust when ad skipping also becomes an option. We adopt the elastic demand model as in Section 4.1 of the paper and assume $c^b = c^s = c$ to eliminate the impacts from the cost difference in ad avoidance methods. We find that our main results from the previous sections remain qualitatively unchanged with ad skipping. In addition, the following two propositions are obtained.

Proposition SA1 *With endogenous d and ad skipping, there is a separating equilibrium that satisfies the Intuitive Criterion with $\mu(s, p_1) = 1$ if $s \geq s^*$ and $p_1 = v$; and $\mu(s, p_1) = 0$ if $s < s^*$ and/or $p_1 \neq v$.*

Let $\bar{k} = \sqrt{\frac{4\bar{\gamma}}{(1-\beta)t}}$ and $\bar{\bar{c}} = \frac{(1-\beta)k^2v^2}{4\bar{\gamma}} - v^2$. If $k \le \bar{k}$, then $s^* = 0$ and d^* is determined by the binding IC_H condition. If $k > \bar{k}$, then $s^* = 0$ and d^* is determined by the binding IC_H condition when $c > \bar{\bar{c}}$; and $s^* = \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2 > 0$ and d^* is determined by the binding IC_L condition when $c \le \bar{\bar{c}}$.

When c changes from $\bar{\bar{c}}^+$ to $\bar{\bar{c}}^-$, the equilibrium jumps discretely from $s^* = 0$ to $s^* > 0$, the total ad spending S^* and the profit of H firm increase, and the number of informed and uninformed consumers who block ad decreases. The profit of the platform is continuous in c at $c = \bar{\bar{c}}$.

Proof of Proposition SA1

(i) We divide the proof into two steps. In step 1, we derive the optimal consumer and firm decisions under ad skipping. In step 2, we derive the platform profit under ad skipping and compare it with the platform profit under ad blocking.

Step 1: Derive optimal consumer and firm decisions

We first derive the equilibrium under ad skipping, starting with consumer decision. If an informed consumer skips an ad right away (Option 1), her utility is

$$u_{s1} = \alpha \cdot (v^2 - c).$$

Note that ad skipping cost c is incurred only when there is an ad, i.e., the firm is of H-type (with probability α).

If the consumer skips ad after seeing s (Option 2), her utility is

$$u_{s2} = \alpha \cdot (v^2 - t \cdot (\gamma - h(s)) - c).$$

If the consumer never skips ad (Option 3), then her utility is

$$u_{s3} = \alpha \left(v^2 - (\gamma - h(s))\right).$$

For informed consumers, Option 2 is strictly dominated by Option 1. So they will either skip ad right away or not skip at all. The marginal consumer is characterized by

$$u_{s1} = u_{s3} \Rightarrow \tilde{\gamma}_I^s = c + h(s).$$

The superscript 's' in $\tilde{\gamma}_I^s$ indicates that this is under ad skipping (instead of ad blocking).

If an uninformed consumer skips ad right away (before learning s), we assume that she remains unaware of the product and thus won't buy. This is similar to ad blocking. Her utility is

$$u_{s1} = -\alpha \cdot c.$$

Similarly, her utility from Option 2 is

$$u_{s2} = \alpha(v^2 - t \cdot (\gamma - h(s)) - c).$$

If the consumer never skips ad, then her utility is

$$u_{s3} = \alpha \left(v^2 - (\gamma - h(s))\right).$$

The distinction between Options 2 and 3 is irrelevant for the firm and the platform, since H-type firm makes sales to the uninformed consumer under both options. For Option 1 to be optimal, it must be that

$$u_{s1} > \max\{u_{s2}, u_{s3}\} \Rightarrow \gamma > \tilde{\gamma}^s_{NI} = \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\},$$

where $\tilde{\gamma}^s_{NI}$ is the marginal consumer who will skip ad right away (and will not purchase).

We now move on to firm decision. H-type firm's profit when advertising is

$$\pi^{ad}_H = \beta(2v^2) + (1-\beta)F(\tilde{\gamma}^s_{NI})(2v^2) - S,$$

where $S = s + d \cdot size^s$ and $size^s = \beta F(\tilde{\gamma}^s_I) + (1-\beta)F(\tilde{\gamma}^s_{NI})$.

Similarly, its profit when not advertising is

$$\pi^{not\_ad}_H = \beta \cdot (2v^2).$$

IC_H is given by

$$\pi^{ad}_H - \pi^{not\_ad}_H = (1-\beta)F(\tilde{\gamma}^s_{NI})(2v^2) - S \geq 0, \qquad (IC_H),$$

where

$$\tilde{\gamma}^s_{NI} = \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\}.$$

L-type firm's profit when advertising is

$$\pi^{ad}_L = (1-\beta)F(\tilde{\gamma}^s_{NI})(v^2) - S,$$

and its profit when not advertising is

$$\pi^{not\_ad}_L = 0.$$

IC_L is given by

$$\pi^{ad}_L - \pi^{not\_ad}_L = (1-\beta)F(\tilde{\gamma}^s_{NI})(v^2) - S \leq 0. \qquad (IC_L)$$

Step 2: Derive optimal platform decision

Recall that $\tilde{\gamma}^s_{NI} = \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\}$. Let Case A denote the case of $\frac{v^2}{t} + h(s) \geq v^2 + c + h(s)$, which is equivalent to

$$c \leq \bar{c} = \frac{v^2}{t} - v^2. \tag{SA.1}$$

Let case B denote the opposite case, i.e., $\frac{v^2}{t} + h(s) < v^2 + c + h(s)$.

With two cases (A and B) and two equilibrium types (1 and 2), there are a total of 4 possible platform profit functions D_i^j, where $i = 1, 2$ denote equilibrium type and $j = A, B$ denote cases A and B respectively.[1] Then

$$\tilde{\gamma}_1^A = \frac{v^2}{t} + h(s), \quad \tilde{\gamma}_1^B = v^2 + c + h(s), \quad \tilde{\gamma}_2^A = \frac{v^2}{t}, \quad \tilde{\gamma}_2^B = v^2 + c,$$
$$D_1^A = \max_{s>0}\ (1-\beta)F(\tilde{\gamma}_1^A) \cdot v^2 - s, \quad D_1^B = \max_{s>0}\ (1-\beta)F(\tilde{\gamma}_1^B) \cdot v^2 - s,$$
$$D_2^A = (1-\beta)F(\tilde{\gamma}_2^A) \cdot (2v^2), \quad D_2^B = (1-\beta)F(\tilde{\gamma}_2^B) \cdot (2v^2).$$

FOCs for D_1^A and D_1^B end up being the same,

$$(1-\beta)\frac{h'(s)}{\bar{\gamma}} \cdot v^2 = s \Rightarrow \frac{1-\beta}{\bar{\gamma}} \cdot k \cdot \frac{1}{2} \cdot s^{-1/2} \cdot v^2 = 1$$
$$\Rightarrow s^* = \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$
$$\Rightarrow h(s^*) = k\sqrt{s^*} = \frac{(1-\beta)k^2v^2}{2\bar{\gamma}},$$

Obviously s^* and $h(s^*)$ are independent of c and are the same in D_1^A and D_1^B. Then

$$\tilde{\gamma}_2^B - \tilde{\gamma}_2^A = \tilde{\gamma}_1^B - \tilde{\gamma}_1^A \Rightarrow D_2^B - D_2^A = 2(D_1^B - D_1^A)$$
$$\Rightarrow D_2^B - D_1^B = (D_2^A - D_1^A) + (D_1^B - D_1^A).$$

Note that

$$D_i^A \geq D_i^B \Leftrightarrow c \leq \bar{c}. \tag{SA.2}$$

Next, we compare D_1^A with D_2^A and compare with D_1^B with D_2^B.

In type 1 equilibrium ($s^* > 0$), substituting the s^* and $h(s^*)$ expressions, we can obtain

$$\begin{aligned} D_1^A &= (1-\beta)\frac{h(s^*) + \frac{v^2}{t}}{\bar{\gamma}} \cdot v^2 - s^* \\ &= (1-\beta)\frac{v^2}{t\bar{\gamma}} \cdot v^2 + \left((1-\beta)\frac{h(s^*)}{\bar{\gamma}} \cdot v^2 - s^*\right) \\ &= (1-\beta)\frac{v^2}{t\bar{\gamma}} \cdot v^2 + \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2, \end{aligned}$$

[1] Note that platform earns profit only when the firm is of H-type, which occurs with probability α. To ease on notation, we ignore the α term which enters into all 4 D_i^j expressions the same way and thus won't affect the comparison of them.

$$D_1^B = (1-\beta)\frac{v^2+c}{\bar{\gamma}} \cdot v^2 + \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2.$$

Similarly, in type 2 equilibrium ($s^* = 0$),

$$D_2^A = (1-\beta)\frac{v^2}{t \cdot \bar{\gamma}} \cdot (2v^2), \quad D_2^B = (1-\beta)\frac{v^2+c}{\bar{\gamma}} \cdot (2v^2).$$

$$D_1^A = D_2^A \Rightarrow (1-\beta)\frac{v^2}{t\bar{\gamma}} \cdot v^2 = \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2$$
$$\Rightarrow k = \bar{k} \equiv \sqrt{\frac{4\bar{\gamma}}{(1-\beta)t}}.$$

Then

$$D_1^A \geq D_2^A \Leftrightarrow k \geq \bar{k}. \tag{SA.3}$$

Similarly,

$$D_1^B = D_2^B \Rightarrow (1-\beta)\frac{v^2+c}{\bar{\gamma}} \cdot v^2 = \left(\frac{(1-\beta)kv^2}{2\bar{\gamma}}\right)^2$$
$$\Rightarrow c = \bar{\bar{c}} \equiv \frac{(1-\beta)k^2v^2}{4\bar{\gamma}} - v^2.$$

Then

$$D_1^B \geq D_2^B \Leftrightarrow c \leq \bar{\bar{c}}. \tag{SA.4}$$

Next, we rank $\bar{c}$ and $\bar{\bar{c}}$, using the conditions in equations (SA.2) – (SA.4). Recall that

$$D_2^B - D_1^B = (D_2^A - D_1^A) + (D_1^B - D_1^A).$$

At $c = \bar{c}$, we have $D_1^A = D_1^B$. Then $D_2^B - D_1^B = D_2^A - D_1^A$. If $k \geq \bar{k}$, then $D_1^A \geq D_2^A$, which implies $D_2^B - D_1^B \leq 0$, evaluated at $c = \bar{c}$. Since $D_1^B \geq D_2^B$ if and only if $c \leq \bar{\bar{c}}$, $\bar{c} \leq \bar{\bar{c}}$ must hold. On the other hand, if $k < \bar{k}$, then $D_1^A < D_2^A$, which implies $D_2^B - D_1^B > 0$, evaluated at $c = \bar{c}$. Since $D_1^B > D_2^B$ if and only if $c < \bar{\bar{c}}$, then $\bar{c} > \bar{\bar{c}}$ must hold. Combined, $\bar{c} \leq \bar{\bar{c}}$ if and only if $k \geq \bar{k}$.

Next, we proceed to characterize the equilibrium.

- Suppose that $k \geq \bar{k}$. Then $\bar{c} \leq \bar{\bar{c}}$ and $D_1^A \geq D_2^A$.
  - If $c \geq \bar{\bar{c}}$, then D_2^B is the maximum platform profit.
  - If $c \in (\bar{c}, \bar{\bar{c}})$, then D_1^B is the maximum platform profit.
  - If $c \leq \bar{c}$, then D_1^A is the maximum.

    Equilibrium jumps from type 2 to type 1 when c decreases from $\bar{\bar{c}}^+$ to $\bar{\bar{c}}^-$. H type advertiser and consumers are all better off and platform profit is continuous.

- Suppose that $k < \bar{k}$. Then $\bar{c} > \bar{\bar{c}}$ and $D_1^A < D_2^A$.
  - If $c \geq \bar{c}$, then D_2^B is the maximum platform profit.
  - If $c \leq \bar{c}$, then D_2^A is the maximum.

    Note that in this case, equilibrium stays in Type 2 ($s^* = 0$).

When $k \geq \bar{k}$ and c decreases from $\bar{\bar{c}}^+$ to $\bar{\bar{c}}^-$, the equilibrium jumps discretely from $s^* = 0$ to $s^* > 0$. Thus the number of informed and uninformed consumers who block ad decreases. Total ad spending increases from D_2 to $D_1 + s^*$ with $s^* > 0$ and $D_1 = D_2$. H-type firm's profit goes up because IC_H changes from binding to slack. The profit of the platform is continuous in c at $c = \bar{\bar{c}}$ ($D_1 = D_2$ is how $c = \bar{\bar{c}}$ is defined).■

Proposition SA2 *The platform prefers ad skipping to ad blocking if and only if* $t < \bar{t} = \frac{v^2}{v^2 + \frac{c}{\alpha}}$.

Proof of Proposition SA2

We compare platform profits under ad skipping and under ad blocking, for both type 1 and type 2 equilibrium.

Let us start with type 1 equilibrium: $s^* > 0$ with IC_L binding. This is the case where d is small so that $s^* > 0$ is required for IC_L to be binding. For any $s > 0$, platform's profits under ad blocking and under ad skipping are

$$\pi^b = (1-\beta)F(\tilde{\gamma}_{NI}^b) \cdot v^2 - s, \quad \pi^s = (1-\beta)F(\tilde{\gamma}_{NI}^s) \cdot v^2 - s,$$

where

$$\tilde{\gamma}_{NI}^b = v^2 + \frac{c}{\alpha} + h(s), \quad \tilde{\gamma}_{NI}^s = \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\}.$$

For any given s, platform profit is higher under ad blocking if and only if

$$\tilde{\gamma}_{NI}^b > \tilde{\gamma}_{NI}^s \Leftrightarrow v^2 + \frac{c}{\alpha} + h(s) > \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\}$$

$$\Leftrightarrow t > \frac{v^2}{v^2 + \frac{c}{\alpha}}.$$

Next, consider type 2 equilibrium where $s^* = 0$ with IC_H binding. When not advertising, H-type firm makes the same profit under ad blocking and under ad skipping. With IC_H binding, platform's profit is higher if and only if H-type firm's profit under advertising (before subtracting S) is higher.

Platform profit under ad blocking (before subtracting S) is

$$\pi^b = \beta \cdot (2v^2) + (1-\beta) \cdot F(\tilde{\gamma}_{NI}^b) \cdot (2v^2),$$

where $\tilde{\gamma}_{NI}^b = \left(v^2 + \frac{c}{\alpha}\right)$, evaluated at $s = 0$.

Similarly, platform profit under ad skipping (before subtracting S) is

$$\pi^s = \beta \cdot (2v^2) + (1-\beta)F(\tilde{\gamma}^s_{NI})(2v^2),$$

where $\tilde{\gamma}^s_{NI} = \max\{\frac{v^2}{t}, (v^2+c)\}$.

Platform profit is higher under ad blocking if and only if

$$\tilde{\gamma}^b_{NI} > \tilde{\gamma}^s_{NI} \Leftrightarrow t > \frac{v^2}{v^2 + \frac{c}{\alpha}},$$

which is the same as the condition in type 1 equilibrium.■

Note that the v^2 term in both the numerator and denominator of the condition in Proposition SA2 comes from the uninformed consumer's surplus gained from a product purchase. If we adopt the inelastic demand model this term will disappear so that $\bar{t} = 0$. Therefore, the profit of the platform is always higher with ad blocking than with ad skipping in the case of inelastic demand.

SA1.4.1 What if the advertiser does not pay under Option 2 above?

This setup is similar to ad skipping analyzed above except that here advertiser does not pay ad distribution cost for consumers take Option 2, i.e., watch the ad briefly to infer s and then skip the ad.

There is no change to the behavior of informed consumers so $\tilde{\gamma}_I = c + h(s)$ continues to hold. If an uninformed consumer skips ad right away before learning s, i.e., Option 1, her utility is

$$u_{s1} = -\alpha \cdot c.$$

Similarly, her utility from Option 2 is

$$u_{s2} = \alpha(v^2 - t \cdot (\gamma - h(s)) - c).$$

If the consumer never skips ad, then her utility is

$$u_{s3} = \alpha\left(v^2 - (\gamma - h(s))\right).$$

Previously the distinction between Options 2 and 3 is irrelevant for firms and advertisers. This is because under either option, the firm makes sales to the uninformed consumers, and always pays for ad distribution cost. However, now the firm pays the platform ad distribution cost for uninformed consumers who take Option 3 but not for those who take Option 2. Nevertheless, we show that this distinction has no impact on our results.

We first derive marginal uninformed consumer, $\tilde{\gamma}_{NI}$, as follows,

$$u_{s1} > \max\{u_{s2}, u_{s3}\} \Rightarrow \gamma > \tilde{\gamma}_{NI} = \max\left\{\frac{v^2}{t} + h(s), v^2 + c + h(s)\right\},$$

where $\tilde{\gamma}_{NI}$ is the marginal consumer who will skip ad right away (and will not purchase).

H-type firm's profit when advertising is

$$\pi_H^{ad} = \beta(2v^2) + (1-\beta)F(\tilde{\gamma}_{NI})(2v^2) - S,$$

where $S = s + d \cdot size$ and $size$ does not include the uninformed consumers who take Option 2. Similarly, its profit when not advertising is

$$\pi_H^{not\_ad} = \beta \cdot (2v^2).$$

Then IC_H is given by

$$\pi_H^{ad} - \pi_H^{not\_ad} = (1-\beta)F(\tilde{\gamma}_{NI})(2v^2) - S \geq 0, \qquad (IC_H).$$

L-type firm's profit when advertising is

$$\pi_L^{ad} = (1-\beta)F(\tilde{\gamma}_{NI})(v^2) - S,$$

and its profit when not advertising is

$$\pi_L^{not\_ad} = 0.$$

Then IC_L is given by

$$\pi_L^{ad} - \pi_L^{not\_ad} = (1-\beta)F(\tilde{\gamma}_{NI2})(v^2) - S \leq 0. \qquad (IC_L)$$

Next, we analyze platform profit. In Case 1 (IC_L binding with $s^* > 0$), we have

$$D_1 = \alpha \cdot \left[(1-\beta)F(\tilde{\gamma}_{NI}(s>0))(v^2) - s\right].$$

In Case 2 (IC_H binding with $s^* = 0$), we have

$$D_2 = \alpha \cdot (1-\beta)F(\tilde{\gamma}_{NI}(s=0))(2v^2).$$

Note that $size$ does not enter D_1 or D_2. This is because any change in $size$ (e.g., due to advertiser not paying for uninformed consumers who take Option 2) is exactly offset by an adjustment in d. Therefore, our previous analysis (for the situation where advertiser pays for uninformed consumers who take Option 2) directly applies to the situation here.

SA2 Alternative rule on equilibrium selection

In the paper, we assume that $\beta > \bar{\beta}$, which ensures that there is no pooling equilibrium so that there will be a unique equilibrium in the subgame given d. Otherwise, the optimal d cannot be determined. Here we show that our main results in the paper do not change if we assume that the equilibrium (either pooling or separating) with higher profit to the H-type firm will be selected if both pooling and separating equilibrium exist. This is because, as detailed below, the H-type firm always earns a higher profit under the separating equilibrium than under the pooling equilibrium. Thus, only the separating equilibrium will be played given the equilibrium selection rule.

Recall that H-type firm's profit in the pooling equilibrium is,

$$\pi_H^{pooling} = \beta(\alpha v + v) + (1-\beta)\cdot F(c)\cdot(\alpha v + v) - d\cdot F(c).$$

Note that firms will always choose $s^* = 0$ in the pooling equilibrium.

In the separating equilibrium, H-type firm's profit is

$$\pi_H^{separating} = \beta(2v) + (1-\beta)F(\tilde{\gamma})(2v) - s - d\cdot F(\tilde{\gamma}),$$

where $\tilde{\gamma} = \frac{c}{\alpha} + h(s)$.

There are two cases in the separating equilibrium, depending on whether $s^* > 0$. We start with case 2, where $s^* = 0$ and IC_H is binding. This occurs when $d \in [(1-\beta)v, (1-\beta)(2v)]$. Then

$$\pi_H^2 = \beta(2v) + (1-\beta)F(c/\alpha)\cdot(2v) - d\cdot F(c/\alpha).$$

Then

$$\begin{aligned}
\pi_H^2 - \pi_H^{pooling} &= \beta(1-\alpha)v + F(c/\alpha)[(1-\beta)(2v) - d] - F(c)[(1-\beta)(1+\alpha)v - d] \\
&> F(c/\alpha)[(1-\beta)(2v) - d] - F(c)[(1-\beta)(2v) - d] \\
&> [F(c/\alpha) - F(c)]\cdot[(1-\beta)(2v) - d] \\
&> 0,
\end{aligned}$$

because $c/\alpha > c$ and $(1-\beta)(2v) > d$. The latter is required to be in case 2.

Since IC_H is binding in case 2 but slack in case 1, H-type firm's profit under case 1 is always (weakly) higher than under case 2. Then $\pi_H^{separating} > \pi_H^{pooling}$ always holds.